(1)

5 **I** Eq diff $\quad y''(x) - 2y'(x) + 2y(x) = \cos x$

SSSM $\quad \Rightarrow r^2 - 2r + 2 = 0 \qquad \Delta = 4 - 8 = (2j)^2$

$$r = \frac{2 \pm 2j}{2} = 1 \pm j$$

$$y(x) = A e^{(1+j)x} + B e^{(1-j)x} = e^{x}\left(A e^{jx} + B e^{-jx}\right)$$
$$= e^{x}(a\cos x + b\sin x) \quad 2$$

SPE $\quad y(x) = c\sin x + d\cos x$ 1 $\Rightarrow y'(x) = c\cos x - d\sin x$

$$\Rightarrow y''(x) = -c\sin x - d\cos x$$

$$\Rightarrow (-c\sin x - d\cos x) - 2(c\cos x - d\sin x) + 2(c\sin x + d\cos x) = \cos x$$

$$\begin{cases} -c + 2d + 2c = 0 \\ -d - 2c + 2d = 1 \end{cases} \Leftrightarrow \begin{cases} c + 2d = 0 \\ -2c + d = 1 \end{cases} \Leftrightarrow \begin{cases} c = -2/5 \\ d = 1/5 \end{cases}$$

finalement $\quad y(x) = e^{x}(a\cos x + b\sin x) + \frac{1}{5}\cos x - \frac{2}{5}\sin x \quad$ 2

II TL / Condensateur

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1) $Q(t) \supset q(p)$

$$\frac{dQ}{dt} \supset p\,q(p) - \overset{\text{CI}}{\underset{''}{Q(0)}} = p\,q(p) \;\; 1 \quad \text{et} \quad \frac{d^2Q}{dt^2} \supset p^2 q(p) - p\,\underbrace{\dot{Q}(0) - Q(0)}_{\text{CI} \; = 0} \;\; 1$$

$$E(t) \supset \int_0^{T_0} \frac{E_0}{T_0} t e^{-pt} dt = \frac{E_0}{T_0}\left(-\frac{T_0}{p} e^{-pT_0} - \frac{1}{p^2} e^{-pT_0} + \frac{1}{p^2}\right) \;\; 2$$

$$\text{eq diff} \Rightarrow \left(Lp^2 + \frac{1}{C}\right) q(p) = \frac{E_0}{T_0}\left(\frac{1}{p^2}(1 - e^{-pT_0}) - \frac{T_0}{p} e^{-pT_0}\right)$$

$$\text{donne} \quad q(p) = \frac{E_0 C}{T_0}\left(\frac{1 - e^{-pT_0}}{p^2(LCp^2+1)} - \frac{T_0 e^{-pT_0}}{p(LCp^2+1)}\right) \;\; 1$$

2) $\dfrac{1}{p^2(\tau^2 p^2 + 1)} = \dfrac{1}{p^2} - \dfrac{\tau^2}{\tau^2 p^2 + 1}$ 1,5 et $\dfrac{1}{p(\tau^2 p^2 + 1)} = \dfrac{1}{p} - \dfrac{\tau^2 p}{(\tau^2 p^2 + 1)}$ 1,5

3) $\tau = \sqrt{LC}$

$$q(p) = \frac{E_0 C}{T_0}\left(\frac{1}{p^2}(1 - e^{-pT_0}) - \frac{\tau^2(1 - e^{-pT_0})}{\tau^2 p^2 + 1} - \frac{T_0 e^{-pT_0}}{p} + \frac{T_0 \tau^2 p\, e^{-pT_0}}{\tau^2 p^2 + 1}\right.$$

$$\text{avec} \quad \sin(t/\tau) \supset \frac{\tau}{\tau^2 p^2 + 1} \quad \text{et} \quad \cos(t/\tau) \supset \frac{\tau^2 p}{\tau^2 p^2 + 1}$$

1

$$\Rightarrow Q(t) = \frac{E_0 C}{T_0}\Big(t H(t) - (t - T_0) H(t - T_0) - \tau H(t)\sin(t/\tau) + \tau H(t - T_0)\sin\left(\frac{t - T_0}{\tau}\right) - T_0 H(t - T_0) + T_0 H(t - T_0)\cos\Big(\frac{t-}{\tau}$$

2 intervalles

$0 < t < T_0$: $\Rightarrow H(t) = 1$ et $H(t-T_0) = 0$

$$\Rightarrow Q(t) = \frac{E_0 C}{T_0}\left(t - \sqrt{LC}\,\sin\left(\frac{t}{\sqrt{LC}}\right)\right) \quad 1$$

$t > T_0$: $H(t) = 1$ et $H(t-T_0) = 1$

$$\Rightarrow Q(t) = \frac{E_0 C}{T_0}\left(-\sqrt{LC}\,\sin\left(\frac{t}{\sqrt{LC}}\right) + \sqrt{LC}\,\sin\left(\frac{t-T_0}{\sqrt{LC}}\right) + T_0\cos\left(\frac{t-T_0}{\sqrt{LC}}\right)\right) \quad 1$$

III

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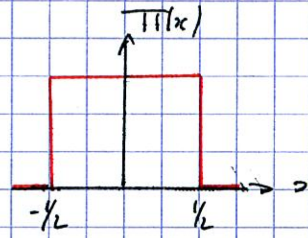

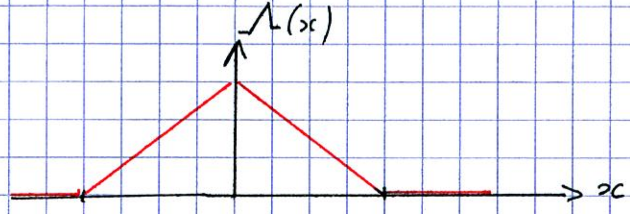

1) TF de $\Pi(x)$

$$F(u) = \int_{-1/2}^{+1/2} 1 \times e^{-i2\pi u x}\,dx$$

$$= \left[\frac{e^{-i2\pi u x}}{-i2\pi u}\right]_{-1/2}^{1/2} = -\frac{1}{i2\pi u}\left(e^{-i\pi u} - e^{i\pi u}\right)$$

$$= \frac{\sin(\pi u)}{\pi u} \quad 1$$

TF de $\Lambda(x)$

$$F(u) = \int_{-1}^{0} (1+x)e^{-i2\pi u x}\,dx + \int_{0}^{1} (1-x)e^{-i2\pi u x}\,dx \quad 1$$

$$= \int_{-1}^{+1} e^{-i2\pi u x}\,dx + \int_{-1}^{0} x e^{-i2\pi u x}\,dx - \int_{0}^{1} x e^{-i2\pi u x}\,dx$$

$$= \frac{\sin(2\pi u)}{\pi u} - \frac{e^{i2\pi u}}{i2\pi u} - \frac{1}{(i2\pi u)^2}\left(1 - e^{i2\pi u}\right)$$

$$- \left(\frac{-e^{-i2\pi u}}{i2\pi u} - \frac{1}{(i2\pi u)^2}\left(e^{-i2\pi u} - 1\right)\right)$$

$$= \frac{2\left(\cos(2\pi u) - 1\right)}{(i2\pi u)^2} = \frac{2\left(\cos^2(\pi u) - \sin^2(\pi u) - 1\right)}{(i2\pi u)^2}$$

$$= \frac{-4\sin^2 \pi u}{-4(\pi u)^2} = \operatorname{sinc}^2(\pi u) \quad 1$$

2) $$\int f'(x)\varphi(x)\,dx = -\int f(x)\varphi'(x)\,dx.$$

$$\int_{-1}^{+1} \Lambda'(x)\varphi(x)\,dx = -\int_{-1}^{+1} \Lambda(x)\varphi'(x)\,dx = -\left(\int_{-1}^{0}(1+x)\varphi'(x)\,dx + \int_{0}^{1}(1-x)\varphi'(x)\,dx\right)$$

$$= -\left(\int_{-1}^{0}\varphi'(x)\,dx + \int_{-1}^{0}x\varphi'(x)\,dx + \int_{0}^{1}\varphi'(x)\,dx - \int_{0}^{1}x\varphi'(x)\,dx\right)$$

$$= -\underbrace{\int_{-1}^{+1}\varphi'(x)\,dx}_{=0} - \left(\left[x\varphi(x)\right]_{-1}^{0} - \int_{-1}^{0}\varphi(x)\,dx\right) + \left(\left[x\varphi(x)\right]_{0}^{1} - \int_{0}^{1}\varphi(x)\,dx\right)$$

$$= -\varphi(-1) + \int_{-1}^{0}\varphi(x)\,dx + \varphi(1) - \int_{0}^{1}\varphi(x)\,dx$$

$$= -\int_{-1}^{0}\delta(x)\varphi(x)\,dx + \int_{-1}^{0}\varphi(x)\,dx + \int_{0}^{1}\delta(x)\varphi(x)\,dx - \int_{0}^{1}\varphi(x)\,dx$$

$$= -\int_{-1}^{0}(\delta(x)-1)\varphi(x)\,dx + \int_{0}^{1}(\delta(x)-1)\varphi(x)\,dx \quad 1$$

$x<0 \quad \Lambda'(x) = 1 = \Pi(x) \quad ; \quad x>0 \quad \Lambda'(x) = -1 = -\Pi(x)$

$$\Rightarrow \Lambda'(x) = \Pi(x+1/2) - \Pi(x-1/2) \quad 1$$

$$TF(\Lambda')(u) = e^{i\pi u}\,TF(\Pi)(u) - e^{-i\pi u}\,TF(\Pi)(u) \quad 1$$

$$= 2i\sin(\pi u)\,TF(\Pi)(u) = 2i\,\frac{\sin^2(\pi u)}{\pi u}$$

or $TF(\Lambda')(u) = 2i\pi u\, TF(\Lambda)(u)$

donc $TF(\Lambda)(u) = \dfrac{TF(\Lambda')(u)}{2i\pi u} = \dfrac{\sin^2(\pi u)}{(\pi u)^2}$ 1 (idem question 1)

4) $$\Pi * \Pi(x) = \int_{-\infty}^{+\infty}\Pi(x-y)\Pi(y)\,dy = \int_{-1/2}^{+1/2}\Pi(x-y)\,dy = -\int_{x+1/2}^{x-1/2}\Pi(v)\,dv \quad 1$$

chgt variable $v = x-y \Rightarrow y = x-v$, $dy = -dv$

$x>1 \quad x-1/2 > 1/2 \Rightarrow \Pi*\Pi(x) = 0$

$x<1 \quad x+1/2 < -1/2 \Rightarrow \Pi*\Pi(x) = 0$

$x \in [0,1]$ $\quad \Pi * \Pi(x) = \int_{x-1/2}^{1/2} \Pi(v)\,dv = \int_{x-1/2}^{1/2} dv = 1 - x$ 0,5

de m $\quad x \in [-1,0]$ $\quad \Pi * \Pi(x) = 1 + x$ 0,5

donc $\quad \Pi * \Pi(x) = \Lambda(x)$

3) $\mathrm{TF}(\Pi * \Pi) = \mathrm{TF}(\Pi) \cdot \mathrm{TF}(\Pi) = \dfrac{\sin(\pi u)}{\pi u} \times \dfrac{\sin(\pi u)}{\pi u} = \dfrac{\sin^2(\pi u)}{(\pi u)^2} = \mathrm{TF}(\Lambda)$ 1

donc $\Lambda = \Pi * \Pi$